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MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

澳門四高校聯合入學考試 (語言科及數學科)

Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2026 試題及參考答案
2026 Examination Paper and Suggested Answer

數學正卷 Mathematics Standard Paper

<u>第一部份</u> **選擇題。請選出每題之最佳答案。**

1. 設集合 $A=\{x:x^2+x-2\le 0\}$，$B=\{x:|x-1|\ge 1\}$，則 $A\cap B=($　　$)$。

   A. $[-2,0]$　　B. $[-2,1]$　　C. $[0,2]$　　D. $[-2,2]$　　E. $(-\infty,+\infty)$

2. 若實數 x 及 y 滿足 $\dfrac{3x+5y}{2y-x}=5$，則 $\dfrac{x}{y}=($　　$)$。

   A. $\dfrac{8}{5}$　　B. $\dfrac{5}{8}$　　C. $\dfrac{15}{8}$　　D. $\dfrac{8}{15}$　　E. $\dfrac{5}{2}$

3. 若關於 x 的方程 $3x-a=5-bx$ 有無數多個解，則 $a-b=($　　$)$。

   A. 2　　B. 8　　C. -2　　D. -8　　E. 15

4. 若二次方程 $x^2-6x+k=0$ 有兩個不相等的實根，則 k 的取值範圍是 $($　　$)$。

   A. $k>36$　　B. $k>9$　　C. $k=9$　　D. $k<36$　　E. $k<9$

5. 若多項式 $x^{1000}+x^{999}+x^{998}+\cdots+x^2+x+1$ 除以 $x+3$，則餘數為 $($　　$)$。

   A. $\dfrac{1}{4}$　　B. $3^{1000}+1$　　C. $\dfrac{1-3^{1001}}{3}$　　D. $\dfrac{1+3^{1001}}{4}$　　E. $\dfrac{1+3^{1001}}{3}$

6. 若 α 和 β 為二次方程 $2x^2-5x+1=0$ 的根，則 $\alpha^2+\beta^2=($　　$)$。

   A. $\dfrac{1}{4}$　　B. $\dfrac{3}{4}$　　C. $\dfrac{21}{4}$　　D. $\dfrac{23}{4}$　　E. $\dfrac{25}{4}$

7. 若 $\log_a(ab)=x$，則 $\log_b(ab)=($　　$)$。

   A. $\dfrac{x}{x+1}$　　B. $\dfrac{x}{x-1}$　　C. $\dfrac{1}{x}$

   D. $\dfrac{x}{1-x}$　　E. 以上皆非

8. 若圓 $x^2+y^2+2x-6y-15=0$ 和直線 ℓ 相切於點 $(-4,-1)$，則直線 ℓ 的方程為 $($　　$)$。

   A. $4x-3y+13=0$　　B. $3x-4y+8=0$　　C. $4x-3y-13=0$

   D. $3x+4y+16=0$　　E. $3x+4y-16=0$

9. 設 $f(x)=ax^3-8x^2+bx-4$，其中 a 和 b 是常數。若 $f(2)=-6$，則 $f(-2)=(\quad)$。

A. 6　　B. 36　　C. 66　　D. -36　　E. -66

10. 若從 6 名男生和 4 名女生中選出 5 人組成一個委員會，其中必須包括至少 3 名男生和至少 1 名女生，則有 (　　) 種不同的組合方式。

A. 180　　B. 200　　C. 240　　D. 252　　E. 280

11. 無理方程 $\sqrt{3x^2-4x+1}-\sqrt{3x^2-2x+4}=-1$ 的解集為 (　　)。

A. $\{0\}$　　B. $\{1\}$　　C. $\{0,2\}$　　D. $\{0,3\}$　　E. 以上皆非

12. 若 $\tan\left(\dfrac{9\pi}{4}+x\right)=-7$，則 $\dfrac{1-\sin 2x}{\cos 2x}=(\quad)$。

A. $-\dfrac{1}{7}$　　B. -7　　C. 2　　D. $\dfrac{1}{5}$　　E. -5

13. 設等比數列 $\{a_n\}_{n=1}^{\infty}$ 的前 n 項和為 S_n。若 $2S_3=a_4-a_1$，$a_3+a_5=7$，則 $a_5+a_7=(\quad)$。

A. 5　　B. 7　　C. 21　　D. 63　　E. 35

14. 設二次函數 $f(x)=-3x^2+2(a-1)x+2$，$a\in\mathbb{R}$。若對任意不同的 x_1，$x_2\in(-1,+\infty)$ 皆有 $(x_1-x_2)(f(x_1)-f(x_2))<0$，則 a 的取值範圍為 (　　)。

A. $[-2,+\infty)$　　B. $(-\infty,-2]$　　C. $[-1,+\infty)$　　D. $(-\infty,-1]$　　E. $[-2,-1]$

15. 設 $f(x)=\begin{cases}\sqrt{|x|}, & |x|\geq 4\\ x, & |x|<4\end{cases}$ 且 $g(x)$ 是二次函數。若複合函數 $f(g(x))$ 的值域是 $[0,+\infty)$，則 $g(x)$ 的值域為 (　　)。

A. $(-\infty,-4]\cup[0,+\infty)$　　B. $(-\infty,-4]\cup[4,+\infty)$　　C. $[0,+\infty)$

D. $[4,+\infty)$　　E. $[-4,+\infty)$

<u>第二部份</u> 解答題。

1. A 和 B 兩隊參加射擊比賽，每隊 3 人，每人射擊一次。擊中目標者為本隊贏得一分，否則零分。假設 A 隊中 3 人擊中目標的概率分別為 $\frac{3}{4}$，$\frac{2}{3}$ 和 $\frac{1}{2}$，B 隊中各人擊中目標的概率均為 $\frac{3}{4}$。假設隊員能否擊中目標相互之間沒有影響。

   (a) 填寫下面表格中 A 隊得分的概率（用最簡分數表示）。 （4 分）

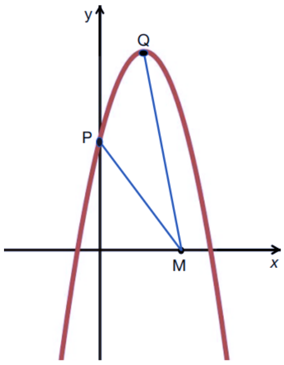
   (b) 求事件“A 和 B 兩隊得分之和等於 3，且 B 隊得分高於 A 隊的得分”的概率。 （4 分）

A 隊得分	0	1	2	3
概率	$\frac{1}{24}$			$\frac{1}{4}$

2. 考慮二次函數 $y = -x^2 + 4x + 5$。該函數圖像與 y 軸的交點為 P 且頂點為 Q。

   (a) 求點 P 和點 Q 的座標。 (3 分)

   (b) 設 M 為 x 軸上任意一點。求 $|PM| + |QM|$ 的最小值，並求此時 M 的座標。 (5 分)



3. 已知 S_n 為等差數列 $\{a_n\}_{n=1}^{\infty}$ 的前 n 項和，$a_5 = 44$ 且 $S_5 = 140$。

   (a) 求數列 $\{a_n\}_{n=1}^{\infty}$ 的通項公式。 (3 分)

   (b) 設 $b_n = \frac{1}{S_n}$，數列 $\{b_n\}_{n=1}^{\infty}$ 的前 n 項和為 T_n。求 T_n 的表達式。 (3 分)

   (c) 證明 $\frac{1}{12} \leq T_n < \frac{3}{16}$ 。 (2 分)

4. 設雙曲線 C: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a > 0, b > 0)$ 過點 $(2\sqrt{3}, 2)$ 且離心率為 $e = \frac{\sqrt{6}}{2}$。斜率為 1 且過點 $(6, 0)$ 的直線 ℓ 與 C 相交與點 A 和點 B。

   (a) 求 C 的方程。 (4 分)

   (b) 若點 M 的座標為 $(2, 0)$，證明直線 AM 和直線 BM 相互垂直。 (4 分)

5. 設函數 $f(x) = \frac{3x}{x+3}$。

(a) (i) 若 $x_1 = \dfrac{3}{2}$，$x_n = f(x_{n-1})$，求 x_2 和 x_3。 (2 分)

(ii) 猜測 $f(x_n)$ 的通項公式並用數學歸納法證明。 (4 分)

(b) 設 $g(x) = \dfrac{f(x^2)}{x}$，$x > 0$。求 $g(x)$ 的最大值。 (2 分)

參考答案

<u>第一部份</u> 選擇題。

題目編號	最佳答案
1	A
2	B
3	C
4	E
5	D
6	C
7	B
8	D
9	E
10	A
11	D
12	A
13	D
14	B
15	C

第二部份 解答題。

1. (a)

A 隊得分	0	1	2	3
概率值	$\frac{1}{24}$	$\frac{1}{4}$	$\frac{11}{24}$	$\frac{1}{4}$

(b) A 得 0 分且 B 得 3 分的概率為 $\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times\left(\frac{3}{4}\right)^3=\frac{9}{512}$，A 得 1 分且 B 得 2 分的概率為 $\left(\frac{3}{4}\times\frac{1}{3}\times\frac{1}{2}+\frac{1}{4}\times\frac{2}{3}\times\frac{1}{2}+\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\right)\times{}_3C_2\left(\frac{3}{4}\right)^2\frac{1}{4}=\frac{27}{256}$。從而事件“A 和 B 兩隊得分之和等於 3，且 B 隊得分高於 A 隊的得分”的概率為 $\frac{9}{512}+\frac{27}{256}=\frac{63}{512}$。

2. (a) 由題意知，$P(0,5)$ 和 $Q(2,9)$。

(b) P 點關於 x-軸的對稱點為 $P'(0,-5)$。直線 $P'Q$ 的方程為 $y=7x-5$。該直線與 x 軸的交點 $M(\frac{5}{7},0)$ 即為所求。$|PM|+|QM|$ 的最小值為 $|P'Q|$，經過計算得 $|P'Q|=10\sqrt{2}$。

3. (a) 由 $a_5=44$ 且 $S_5=140$ 知，$a_1+4d=44$ 和 $\frac{5(a_1+a_5)}{2}=140$，解得 $a_1=12$ 和 $d=8$。因此 $a_n=12+8(n-1)=8n+4$。

(b) 由 $S_n=\frac{n(a_1+a_n)}{2}=\frac{n[24+8(n-1)]}{2}=4n(n+2)$ 知，$b_n=\frac{1}{S_n}=\frac{1}{4n(n+2)}$。於是 $T_n=\sum_{k=1}^{n}\frac{1}{[4k(k+2)]}=\frac{1}{8}\sum_{k=1}^{n}[(\frac{1}{k}-\frac{1}{k+2})]=\frac{1}{8}[\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}]=\frac{3}{16}-\frac{1}{8}[\frac{1}{n+1}+\frac{1}{n+2}]$。

(c) 因為 $\frac{1}{n+1}+\frac{1}{n+2}>0$，所以 $T_n<\frac{3}{16}$。另一方面，$T_1=\frac{1}{S_1}=\frac{1}{a_1}=\frac{1}{12}$，$T_n$ 是關於 n 的單調遞增函數，因此 $T_n\geq T_1=\frac{1}{12}$。

4. (a) 由題意知，$\frac{12}{a^2}-\frac{4}{b^2}=1$ 及 $e=\frac{c}{a}=\frac{\sqrt{6}}{2}$。又因為 $a^2+b^2=c^2$，聯立解得 $a=2$ 和 $b=\sqrt{2}$。因此所求 $\mathcal{C}$ 的方程為 $\dfrac{x^2}{4}-\dfrac{y^2}{2}=1$。

(b) 容易求出直線 ℓ 方程為 $y=x-6$。設交點 $A(x_1,y_1)$ 和 $B(x_2,y_2)$。聯立直線與雙曲線方程的方程組，消除 y，化簡得 $x^2-24x+76=0$。由韋達定理知，$x_1+x_2=24$ 及 $x_1x_2=76$。直線 AM 的斜率 $K_{AM}=\dfrac{y_1}{x_1-2}=\dfrac{x_1-6}{x_1-2}$，直線 BM 的斜率 $K_{BM}=\dfrac{y_2}{x_2-2}=\dfrac{x_2-6}{x_1-2}$。因為

$K_{AM}\times K_{BM}=\dfrac{x_1-6}{x_1-2}\times\dfrac{x_2-6}{x_1-2}=\dfrac{x_1x_2-6(x_1+x_2)+36}{x_1x_2-2(x_1+x_2)+4}=\dfrac{-32}{32}=-1$，所以 AM 和 BM 相互垂直。

5. (a) (i) $x_2=f(x_1)=1, x_3=f(x_2)=\dfrac{3}{4}$。

(ii) 猜測 $x_n=\dfrac{3}{n+1}$。下面用數學歸納法證明：(1) 當 $n=1$ 時，公式成立。(2) 設 $n=k$ 時公式成立，即 $x_k=\dfrac{3}{k+1}$。當 $n=k+1$ 時，$x_{k+1}=f(x_k)=\dfrac{3x_k}{x_k+3}=\dfrac{3\frac{3}{k+1}}{\frac{3}{k+1}+3}=\dfrac{3}{(k+1)+1}$，公式也成立。

(b) $g(x)=\dfrac{f(x^2)}{x}=\dfrac{3x}{x^2+3}$。因為 $x>0$，所以由重要不等式知，$\dfrac{3x}{x^2+3}=\dfrac{3}{x+\frac{3}{x}}\leq\dfrac{\sqrt{3}}{2}$。因此，當 $x=\sqrt{3}$ 時，$g(x)$ 有最大值 $\dfrac{\sqrt{3}}{2}$。

<u>Part I</u> Multiple choice questions. Choose the *best answer* for each question.

1. Let $A = \{x : x^2 + x - 2 \leq 0\}$ and $B = \{x : |x - 1| \geq 1\}$. Then $A \cap B = (\quad)$.

   A. $[-2, 0]$ B. $[-2, 1]$ C. $[0, 2]$ D. $[-2, 2]$ E. $(-\infty, +\infty)$

2. If real numbers x and y satisfy the equation $\dfrac{3x + 5y}{2y - x} = 5$, then $\dfrac{x}{y} = (\quad)$.

   A. $\dfrac{8}{5}$ B. $\dfrac{5}{8}$ C. $\dfrac{15}{8}$ D. $\dfrac{8}{15}$ E. $\dfrac{5}{2}$

3. If the equation in x, $3x - a = 5 - bx$, has infinitely many solutions, then $a - b = (\quad)$.

   A. 2 B. 8 C. -2 D. -8 E. 15

4. If the quadratic equation $x^2 - 6x + k = 0$ has two distinct real roots, then the range of values of k is $(\quad)$.

   A. $k > 36$ B. $k > 9$ C. $k = 9$ D. $k < 36$ E. $k < 9$

5. If the polynomial $x^{1000} + x^{999} + x^{998} + \cdots + x^2 + x + 1$ is divided by $x + 3$, then the remainder is $(\quad)$.

   A. $\dfrac{1}{4}$ B. $3^{1000} + 1$ C. $\dfrac{1 - 3^{1001}}{3}$ D. $\dfrac{1 + 3^{1001}}{4}$ E. $\dfrac{1 + 3^{1001}}{3}$

6. If α and β are the roots of the quadratic equation $2x^2 - 5x + 1 = 0$, then $\alpha^2 + \beta^2 = (\quad)$.

   A. $\dfrac{1}{4}$ B. $\dfrac{3}{4}$ C. $\dfrac{21}{4}$ D. $\dfrac{23}{4}$ E. $\dfrac{25}{4}$

7. If $\log_a(ab) = x$, then $\log_b(ab) = (\quad)$.

   A. $\dfrac{x}{x+1}$ B. $\dfrac{x}{x-1}$ C. $\dfrac{1}{x}$

   D. $\dfrac{x}{1-x}$ E. None of the above

8. If the circle $x^2 + y^2 + 2x - 6y - 15 = 0$ and the line ℓ are tangent at the point $(-4, -1)$, then the equation of the line ℓ is $(\quad)$.

   A. $4x - 3y + 13 = 0$ B. $3x - 4y + 8 = 0$ C. $4x - 3y - 13 = 0$

   D. $3x + 4y + 16 = 0$ E. $3x + 4y - 16 = 0$

9. Let $f(x) = ax^3 - 8x^2 + bx - 4$ with constants a and b. If $f(2) = -6$, then $f(-2) = ($ $)$.

A. 6 B. 36 C. 66 D. -36 E. -66

10. If 5 people are selected from a group of 6 boys and 4 girls to form a committee, with at least 3 boys and at least 1 girl required, then there are ( ) different combinations.

A. 180 B. 200 C. 240 D. 252 E. 280

11. The solution set of the irrational equation $\sqrt{3x^2 - 4x + 1} - \sqrt{3x^2 - 2x + 4} = -1$ is ( ).

A. $\{0\}$ B. $\{1\}$ C. $\{0, 2\}$

D. $\{0, 3\}$ E. None of the above

12. If $\tan\left(\dfrac{9\pi}{4} + x\right) = -7$, then $\dfrac{1 - \sin 2x}{\cos 2x} = ($ $)$.

A. $-\dfrac{1}{7}$ B. -7 C. 2 D. $\dfrac{1}{5}$ E. -5

13. Let S_n be the n-th partial sum of a geometric sequence $\{a_n\}_{n=1}^{\infty}$. If $2S_3 = a_4 - a_1$, $a_3 + a_5 = 7$, then $a_5 + a_7 = ($ $)$.

A. 5 B. 7 C. 21 D. 63 E. 35

14. Let $f(x) = -3x^2 + 2(a-1)x + 2$, $a \in \mathbb{R}$. If, for any distinct x_1, $x_2 \in (-1, +\infty)$, the inequality $(x_1 - x_2)(f(x_1) - f(x_2)) < 0$ holds, then the range of the values of a is ( ).

A. $[-2, +\infty)$ B. $(-\infty, -2]$ C. $[-1, +\infty)$ D. $(-\infty, -1]$ E. $[-2, -1]$

15. Let $f(x) = \begin{cases} \sqrt{|x|}, & |x| \geq 4 \\ x, & |x| < 4 \end{cases}$ and $g(x)$ be a quadratic function. If the range of the composite function $f(g(x))$ is $[0, +\infty)$, then the range of $g(x)$ is ( ).

A. $(-\infty, -4] \cup [0, +\infty)$ B. $(-\infty, -4] \cup [4, +\infty)$ C. $[0, +\infty)$

D. $[4, +\infty)$ E. $[-4, +\infty)$

Part II Problem-solving questions.

1 Teams A and B take part in a shooting competition. Each team has three members, and each member shoots once. A successful hit awards the shooter's team 1 point, while a miss awards 0 points. For Team A, the shooters' hit probabilities are $\frac{3}{4}, \frac{2}{3}$ and $\frac{1}{2}$, respectively. For Team B, each shooter has a hit probability of $\frac{3}{4}$. All shots are assumed to be mutually independent across shooters.

(a) Fill in the probabilities of Team A's score in the table below (in the simplest form). (4 marks)

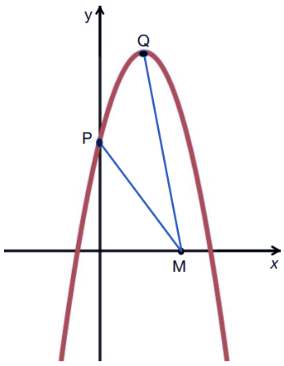
(b) Find the probability of the event that 'the sum of the scores of Teams A and B equals 3 and Team B's score is greater than Team A's score'. (4 marks)

Team A's score	0	1	2	3
Probability	$\frac{1}{24}$			$\frac{1}{4}$

2 Consider the quadratic function $y = -x^2 + 4x + 5$. The point where the graph of the function intersects the y-axis is P and the vertex of the graph is Q.

(a) Find the coordinates of points P and Q. (3 marks)

(b) Let M be an arbitrary point on the x-axis. Find the minimum value of $|PM|+|QM|$ and the coordinates of M where it occurs. (5 marks)



3 Suppose that S_n denotes the n-th partial sum of the arithmetic sequence $\{a_n\}_{n=1}^{\infty}$, $a_5 = 44$ and $S_5 = 140$.

(a) Find the general term formula of the sequence $\{a_n\}_{n=1}^{\infty}$. (3 marks)

(b) Let $b_n = \frac{1}{S_n}$ and let T_n be the n-th partial sum of the sequence $\{b_n\}_{n=1}^{\infty}$. Then find the expression for T_n. (3 marks)

(c) Prove that $\frac{1}{12} \leq T_n < \frac{3}{16}$. (2 marks)

4 Let C be the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a > 0, b > 0)$ passing through $(2\sqrt{3}, 2)$ with eccentricity $e = \frac{\sqrt{6}}{2}$. The line ℓ of slope 1 through $(6, 0)$ intersects C at points A and B.

(a) Find the equation of $\mathcal{C}$. (4 marks)

(b) If point M has coordinates $(2, 0)$, prove that lines AM and BM are perpendicular. (4 marks)

5 Let the function $f(x) = \dfrac{3x}{x+3}$.

(a) (i) If $x_1 = \dfrac{3}{2}$ and $x_n = f(x_{n-1})$, find x_2 and x_3. (2 marks)

(ii) Guess a general formula for $f(x_n)$ and prove it by mathematical induction. (4 marks)

(b) Let $g(x) = \dfrac{f(x^2)}{x}, x > 0$. Find the maximum value of $g(x)$. (2 marks)

Suggested Answer

<u>Part I</u> Multiple-Choice Questions.

Question No.	Best Answer
1	A
2	B
3	C
4	E
5	D
6	C
7	B
8	D
9	E
10	A
11	D
12	A
13	D
14	B
15	C

Part II **Problem-solving questions.**

1. (a)

Team A's score	0	1	2	3
Probability	$\dfrac{1}{24}$	$\dfrac{1}{4}$	$\dfrac{11}{24}$	$\dfrac{1}{4}$

(b) The probability that A scores 0 and B scores 3 is $\frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \left(\frac{3}{4}\right)^3 = \frac{9}{512}$. The probability that A scores 1 and B scores 2 is $\left(\frac{3}{4} \times \frac{1}{3} \times \frac{1}{2} + \frac{1}{4} \times \frac{2}{3} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2}\right) \times {}_3C_2 \left(\frac{3}{4}\right)^2 \frac{1}{4} = \frac{27}{256}$. Hence, the probability of the event "the total score of Teams A and B is 3, and Team B scores more than Team A" is $\frac{9}{512} + \frac{27}{256} = \frac{63}{512}$.

2. (a) From the question, we know that $P(0, 5)$ and $Q(2, 9)$.

(b) The reflection of point P about the x-axis is $P'(0, -5)$. The equation of the line $P'Q$ is $y = 7x - 5$. Its intersection with the x-axis is $M\left(\dfrac{5}{7}, 0\right)$, which is the required point. The minimum value of $|PM| + |QM|$ is $|P'Q|$, and by calculation, $|P'Q| = 10\sqrt{2}$.

3. (a) Since $a_5 = 44$ and $S_5 = 140$, we have $a_1 + 4d = 44$ and $\frac{5(a_1+a_5)}{2} = 140$. Solving equations gives $a_1 = 12$ and $d = 8$. Therefore, $a_n = 12 + 8(n-1) = 8n + 4$.

(b) Since $S_n = \frac{n(a_1+a_n)}{2} = \frac{n[24+8(n-1)]}{2} = 4n(n+2)$, we have $b_n = \frac{1}{S_n} = \frac{1}{4n(n+2)}$. Thus,

$T_n = \sum_{k=1}^{n} \frac{1}{4k(k+2)} = \frac{1}{8} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+2}\right) = \frac{1}{8}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{3}{16} - \frac{1}{8}\left(\frac{1}{n+1} + \frac{1}{n+2}\right).$

(c) Since $\frac{1}{n+1} + \frac{1}{n+2} > 0$, we have $T_n < \frac{3}{16}$. On the other hand, $T_1 = \frac{1}{S_1} = \frac{1}{a_1} = \frac{1}{12}$. As T_n is increasing with respect to n, we have $T_n \geq T_1 = \frac{1}{12}$.

4. (a) From the question, we have $\frac{12}{a^2} - \frac{4}{b^2} = 1$ and $e = \frac{c}{a} = \frac{\sqrt{6}}{2}$. Since $a^2 + b^2 = c^2$, we can obtain $a = 2$ and $b = \sqrt{2}$. Therefore, the equation of $\mathcal{C}$ is $\dfrac{x^2}{4} - \dfrac{y^2}{2} = 1$.

(b) It is easy to find that the equation of the line ℓ is $y = x - 6$. Let the intersection points be $A(x_1, y_1)$ and $B(x_2, y_2)$. Solving the system formed by the line and the hyperbola and eliminating y, we get

$x^2 - 24x + 76 = 0$. By Vieta's formulas, $x_1 + x_2 = 24$ and $x_1 x_2 = 76$.

The slope of line AM is $K_{AM} = \dfrac{y_1}{x_1 - 2} = \dfrac{x_1 - 6}{x_1 - 2}$, and the slope of line BM is $K_{BM} = \dfrac{y_2}{x_2 - 2} = \dfrac{x_2 - 6}{x_2 - 2}$. Hence, $K_{AM} \times K_{BM} = \dfrac{x_1 - 6}{x_1 - 2} \times \dfrac{x_2 - 6}{x_2 - 2} = \dfrac{x_1 x_2 - 6(x_1 + x_2) + 36}{x_1 x_2 - 2(x_1 + x_2) + 4} = \dfrac{-32}{32} = -1$.

Therefore, AM and BM are perpendicular to each other.

5. (a) (i) $x_2 = f(x_1) = 1, x_3 = f(x_2) = \dfrac{3}{4}$.

(ii) We conjecture that $x_n = \dfrac{3}{n+1}$. Now we prove it by mathematical induction:

(1) When $n = 1$, the formula holds.

(2) Assume that the formula holds when $n = k$, that is, $x_k = \dfrac{3}{k+1}$. Then when $n = k + 1$, $x_{k+1} = f(x_k) = \dfrac{3x_k}{x_k + 3} = \dfrac{3 \cdot \frac{3}{k+1}}{\frac{3}{k+1} + 3} = \dfrac{3}{(k+1)+1}$. So the formula also holds for $n = k + 1$.

Therefore, by mathematical induction, $x_n = \dfrac{3}{n+1}$.

(b) $g(x) = \dfrac{f(x^2)}{x} = \dfrac{3x}{x^2 + 3}$. Since $x > 0$, $\dfrac{3x}{x^2+3} = \dfrac{3}{x + \frac{3}{x}} \leq \dfrac{\sqrt{3}}{2}$. Therefore, when $x = \sqrt{3}$, $g(x)$ attains its maximum value $\dfrac{\sqrt{3}}{2}$.